FastTrack - MA109

Exponents and Review of Polynomials

Katherine Paullin, Ph.D.
Lecturer, Department of Mathematics
University of Kentucky
katherine.paullin@uky.edu

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REEF Question

Are you here today? Are you on time?
A) YES
B) NO

Outline

Section 1

Exponents

Exponents

Recall from last class....

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

3 is called the base and 4 is called the exponent.

Today, we will be looking at properties of exponents.

Properties of Exponents

Product Property of Exponents

For any base b and positive integers m and n:

$$b^m \cdot b^n = b^{m+n}$$

Power Property of Exponents

For any base b and positive integers m and n:

$$(b^m)^n = b^{m \cdot n}$$

Examples

Multiply terms using exponential Properties

1. $-4x^3 \cdot \frac{1}{2}x^2$

$$\begin{aligned} -4x^3 \cdot \frac{1}{2}x^2 &= (-4 \cdot \frac{1}{2})(x^3 \cdot x^2) \\ &= (-2)(x^{3+2}) \\ &= -2x^5 \end{aligned}$$

2. $(p^3)^2 \cdot (p^4)^2$

$$\begin{aligned} (p^3)^2 \cdot (p^4)^2 &= p^6 \cdot p^8 \\ &= p^{6+8} \\ &= p^{14} \end{aligned}$$

Properties of Exponents

Product to a Power

For any bases a and b and positive integers m, n, and p:

$$(a^m b^n)^p = a^{mp} b^{np}$$

Quotient to a Power

For any bases a and $b \neq 0$ and positive integers m, n, and p:

$$(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$$

Examples

Simplify using the power property (if possible):

1. $(-3a)^2$

$$\begin{aligned}(-3a)^2 &= (-3)^2 \cdot (a^1)^2 \\ &= 9a^2\end{aligned}$$

2. $-3a^2$ $\quad$ $-3a^2$ is in simplified form.
3. $(\frac{-5a^3}{2b})^2$

$$\begin{aligned}(\frac{-5a^3}{2b})^2 &= \frac{(-5)^2(a^3)^2}{2^2b^2} \\ &= \frac{25a^6}{4b^2}\end{aligned}$$

Properties of Exponents

Quotient Property of Exponents

For any base $b \neq 0$ and positive integers m and n:

$$\frac{b^m}{b^n} = b^{m-n}$$

Property of Negative Exponents

For any base $b \neq 0$ and integer n:

$$\frac{b^{-n}}{1} = \frac{1}{b^n}$$
$$\frac{1}{b^{-n}} = \frac{b^n}{1}$$
$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n; a \neq 0$$

Zero Exponent Property

For any base $b \neq 0$:

$$b^0 = 1$$

Examples

Simplify using exponential Properties. Answer using positive exponents only.

1. $\left(\frac{2a^3}{b^2}\right)^{-2}$

   $\left(\frac{2a^3}{b^2}\right)^{-2} = \left(\frac{b^2}{2a^3}\right)^2 = \left(\frac{(b^2)^2}{2^2(a^3)^2}\right) = \frac{b^4}{4a^6}$

2. $(3x)^0 + 3x^0 + 3^{-2}$

$$\begin{aligned}(3x)^0 + 3x^0 + 3^{-2} &= (1) + 3(1) + \frac{1}{3^2} \\ &= 4 + \frac{1}{9} = \frac{36}{9} + \frac{1}{9} = \frac{37}{9}\end{aligned}$$

Section 2

Polynomials

Definitions

Definition

A **monomial** is a term using only whole number exponents on variables, with no variables in the denominator.
A **polynomial** is a monomial or any sum or difference of monomial terms.

Example	Non-Example
$\frac{1}{2}x^2 - 5x + 6$	$3n^{-2} + 2n - 7$

A polynomial with 2 terms is called a **binomial**.
A polynomial with 3 terms is called a **trinomial**.

Adding and Subtracting Polynomials

To add and subtract polynomials, we use our distributive, commutative, and associative properties to combine like terms.

Example

Compute $(x^3 - 5x + 9) - (x^3 + 3x^2 + 2x - 8)$.

$$\begin{aligned}(x^3 - 5x + 9) - (x^3 + 3x^2 + 2x - 8) &= x^3 - 5x + 9 - x^3 - 3x^2 - 2x + 8 \\ &= (x^3 - x^3) - 3x^2 + (-5x - 2x) \\ &\quad + (9 + 8) \\ &= -3x^2 - 7x + 17\end{aligned}$$

Multiplying Polynomials

To multiply a monomial by a polynomial, use the distributive property.

Example

Find the product. $-2a^2(a^2 - 2a + 1)$

$$\begin{aligned} -2a^2(a^2 - 2a + 1) &= (-2a^2)(a^2) - (-2a^2)(2a) + (-2a^2)(1) \\ &= -2a^4 + 4a^3 - 2a^2 \end{aligned}$$

Multiplying Polynomials

To multiply a binomial by a polynomial, use the distributive property again, distributing each term of the binomial to the polynomial.

Example

Find the product. $(2z + 1)(z - 2)$

$$\begin{aligned}(2z + 1)(z - 2) &= 2z(z - 2) + 1(z - 2) \\ &= 2z^2 - 4z + 1z - 2 \\ &= 2z^2 - 3z - 2\end{aligned}$$

Example

Find the product. $(2v-3)(4v^2+6v+9)$

$$\begin{aligned}(2v-3)(4v^2+6v+9) &= 2v(4v^2+6v+9)-3(4v^2+6v+9)\\ &= 8v^3+12v^2+18v-12v^2-18v-27\\ &= 8v^3-27\end{aligned}$$

Binomial Conjugates

$$(A+B)(A-B) = A^2 - B^2$$

Binomial Squares

$$(A+B)^2 = A^2 + 2AB + B^2$$
$$(A-B)^2 = A^2 - 2AB + B^2$$

Section 3

Practice

Determine the product.

1. $\frac{2}{3}n^2 \cdot 21n^5$
2. $d^2 \cdot d^4 \cdot (c^5)^2 \cdot (c^3)^2$

Simplify the expression.

1. $(6pq^2)^3$
2. $\frac{8z^7}{16z^5}$
3. $(\frac{5p^2q^3r^4}{-2pq^2r^4})^2$

Find the sum or difference.

1. $(3p^3 - 4p^2 + 2p - 7) + (p^2 - 2p - 5)$
2. $(\frac{3}{4}x^2 - 5x + 2) - (\frac{1}{2}x^2 + 3x - 4)$

REEF Question

Find the sum or difference.

1. $(3p^3 - 4p^2 + 2p - 7) + (p^2 - 2p - 5)$

SOLUTIONS

Determine the product.

1. $\frac{2}{3}n^2 \cdot 21n^5$ $14n^7$
2. $d^2 \cdot d^4 \cdot (c^5)^2 \cdot (c^3)^2$ d^6c^{16}

Simplify the expression.

1. $(6pq^2)^3$ $216p^3q^6$
2. $\frac{8z^7}{16z^5}$ $\frac{z^2}{2}$
3. $(\frac{5p^2q^3r^4}{-2pq^2r^4})^2$ $\frac{25p^2q^2}{4}$

Find the sum or difference.

1. $(3p^3 - 4p^2 + 2p - 7) + (p^2 - 2p - 5)$ $3p^3 - 3p^2 - 12$
2. $(\frac{3}{4}x^2 - 5x + 2) - (\frac{1}{2}x^2 + 3x - 4)$ $\frac{1}{4}x^2 - 8x + 6$

Rewriting an Expression

In advanced mathematics, negative exponents are widely used because they are easier to work with than rational expressions. Rewrite the expression $\frac{5}{x^3} + \frac{3}{x^2} + \frac{2}{x^1} + 4$ using negative exponents.

Volume of a Cube

The formula for the volume of a cube is $V = S^3$, where S is the length of one edge. If the length of each edge is $2x^2$:

1. Find a formula for volume in terms of x.
2. Find the volume if $x = 2$.

REEF Question

Volume of a Cube

The formula for the volume of a cube is $V = S^3$, where S is the length of one edge. If the length of each edge is $2x^2$:

1. Find a formula for volume in terms of x.

SOLUTIONS

Rewriting an Expression

In advanced mathematics, negative exponents are widely used because they are easier to work with than rational expressions. Rewrite the expression $\frac{5}{x^3} + \frac{3}{x^2} + \frac{2}{x^1} + 4$ using negative exponents. $5x^{-3} + 3x^{-2} + 2x^{-1} + 4$

Volume of a Cube

The formula for the volume of a cube is $V = S^3$, where S is the length of one edge. If the length of each edge is $2x^2$:

1. Find a formula for volume in terms of x.
   $V = S^3 = (2x^2)^3 = 8x^6$
2. Find the volume if $x = 2$.
   $V = 8x^6 = 8(2)^6 = 8 \cdot 64 = 512 units^3$